QP CODE: 25802571

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Reg No : ....

Name : ....

INTEGRATED MSC DEGREE EXAMINATION, MAY 2025

Fourth Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

COMPLEMENTARY - IPH4CM04 - SPECIAL FUNCTIONS, LA PLACE TRANSFORMS AND COMPLEX ANALYSIS

2021 Admission Onwards

9631B182

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Give Legendre's equation.
2. What is Legendre polynomial of order n?
3. Define the Laplace Transform of $f(t)$. Also find $L(1)$.
4. Find the Laplace Transform of the function $f(t) = 2t + 6$.
5. Find the inverse Laplace Transform of $\frac{2}{s} + \frac{1}{s+2}$
6. Define limit and continuity of a complex function $f(z)$ at $z = z_0$.
7. Find the real and imaginary parts of $\sinh z$ and $\cosh z$.
8. Define the natural logarithm $ln(z)$ of a complex number z.
9. (a) State the Cauchy's integral theorem.
   (b)If $f(z)$ is an entire function, what is the value of its integral over any simple closed curve? Hence what is $\oint_c e^z\, dz$.
10. Integrate $g(z) = \frac{z^2+1}{z^2-1}$ in the counter clockwise direction around a circle of radius 1 with center $z = \frac{1}{2}$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Give the power series expansion of CosX

Turn Over

12. Give the general solution of Bessel's equation for all $x \neq 0$

13. Prove that $L[t^a] = \frac{\Gamma(a+1)}{s^{a+1}}$.

14. Find the Laplace Transform of $t^2\, e^{-3t} sin\, 2t$.

15. Find and plot all the cube roots of $8i$.

16. Prove that $|sinz|^2 = sin^2 x + sinh^2 y$.

17. Using the ML Inequality, prove that $|\int_c \frac{1}{z^4} dz| \leq 4\sqrt{2}$ where C is the line segment from $z = i$ to $z = 1$ and observing that of all the points on this line sgement the mid point is the closest.

18. Integrate the function $f(z) = \frac{e^{3z}}{(4z-\pi i)^3}$ in the counter clockwise direction around the circle $|z| = 1$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. Apply the binomial theorem to $(x^2-1)^n$ and differentiating prove that $P_n(x) = \frac{1}{n!2^n} \frac{d^n}{dx^n}(x^2-1)^n$

20. Evaluate (i) $L^{-1}[\frac{s-a}{s(s+a)}]$.
 (ii) $L^{-1}[\frac{s-2}{s^2(s^2+4)}]$

21. Check whether the function $u = x^2 + y^2$ is harmonic or not. If yes, find a corresponding analytic function $f(z)$.

22. (a) When do we say that integral of a function $f(z)$ is independent of path in a domain D?
 (b) Explain the principle of deformation.
 (c) Show that $\int_c \frac{1}{z} \mathrm{d}z = 2\pi i$ where C is any closed contour oriented in the counter cloclwise direction surrounding the origin.

(2×5=10 weightage)